Non-Self-Embedding Grammars and Descriptional Complexity

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UNIVERSITÀ DEGLI STUDI DI MILANO

DIPARTIMENTO DI INFORMATICA

Non-Self-Embedding Grammars: Recalling the Definition

- Context-Free Grammar $G = (V, \Sigma, P, S)$
- $A \stackrel{*}{\Longrightarrow} \alpha A \beta$
- α, β:
- NSE grammars only generate regular languages [Chomsky, 1959a, Chomsky, 1959b, Anselmo et al., 2002]

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Non-Self-Embedding Grammars to Automata

Step I: The Production Transition Graph

$S \to aA \mid bB \mid cC$
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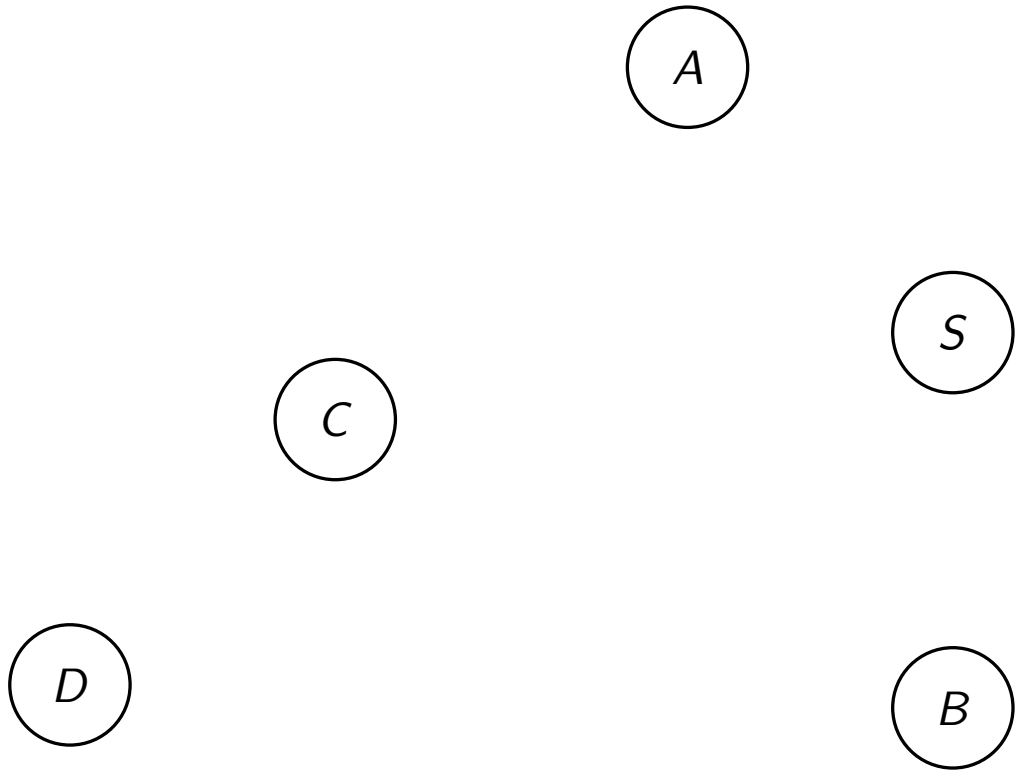

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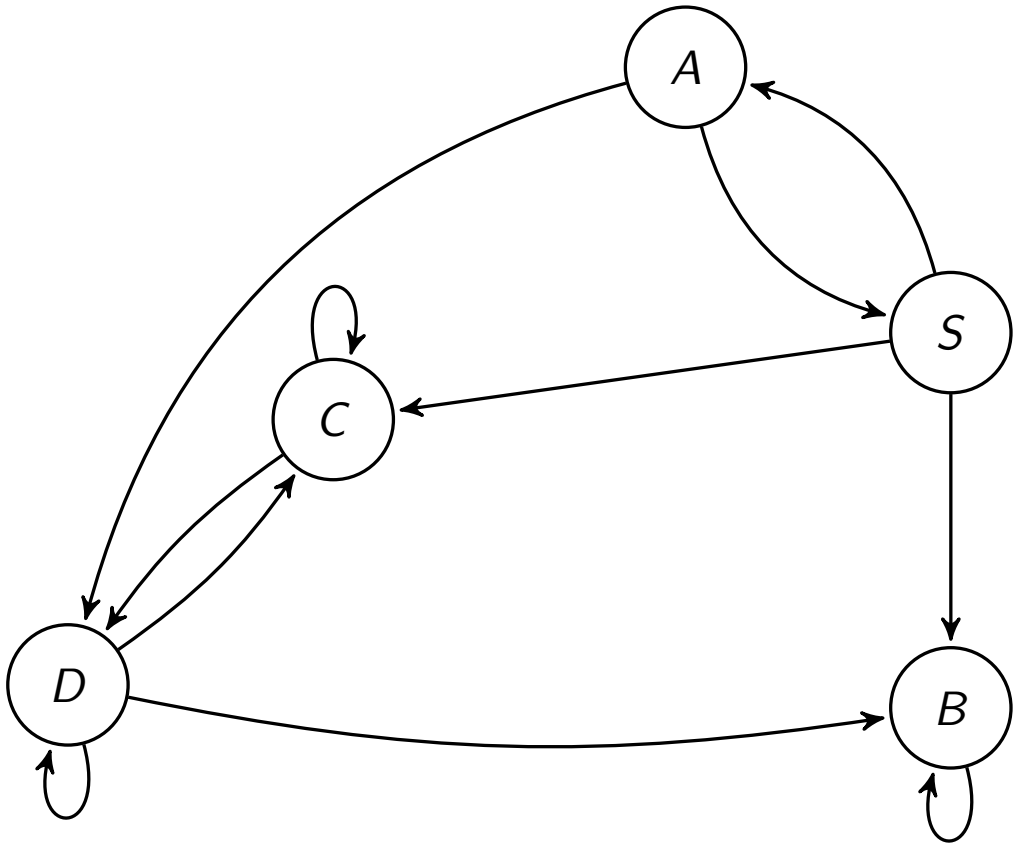

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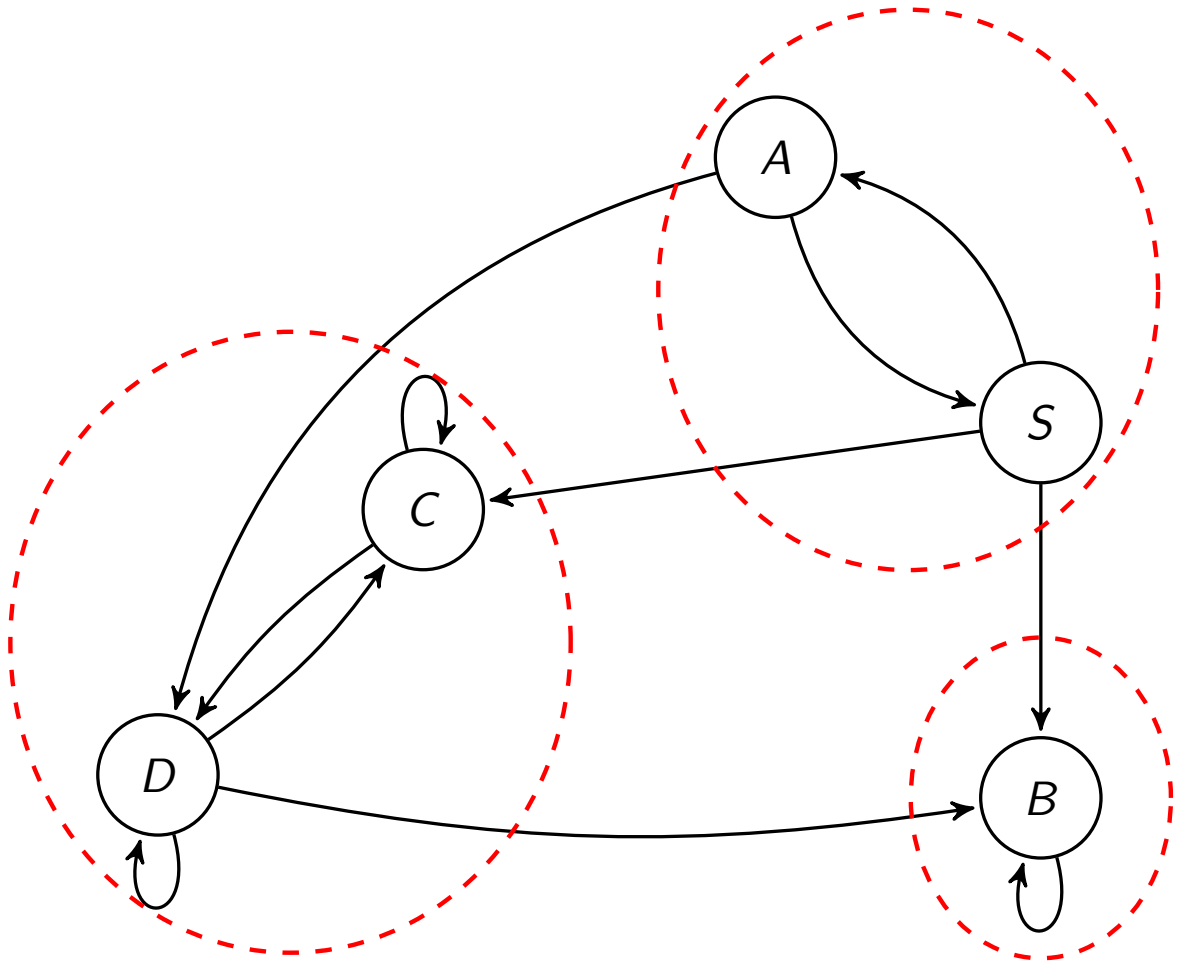

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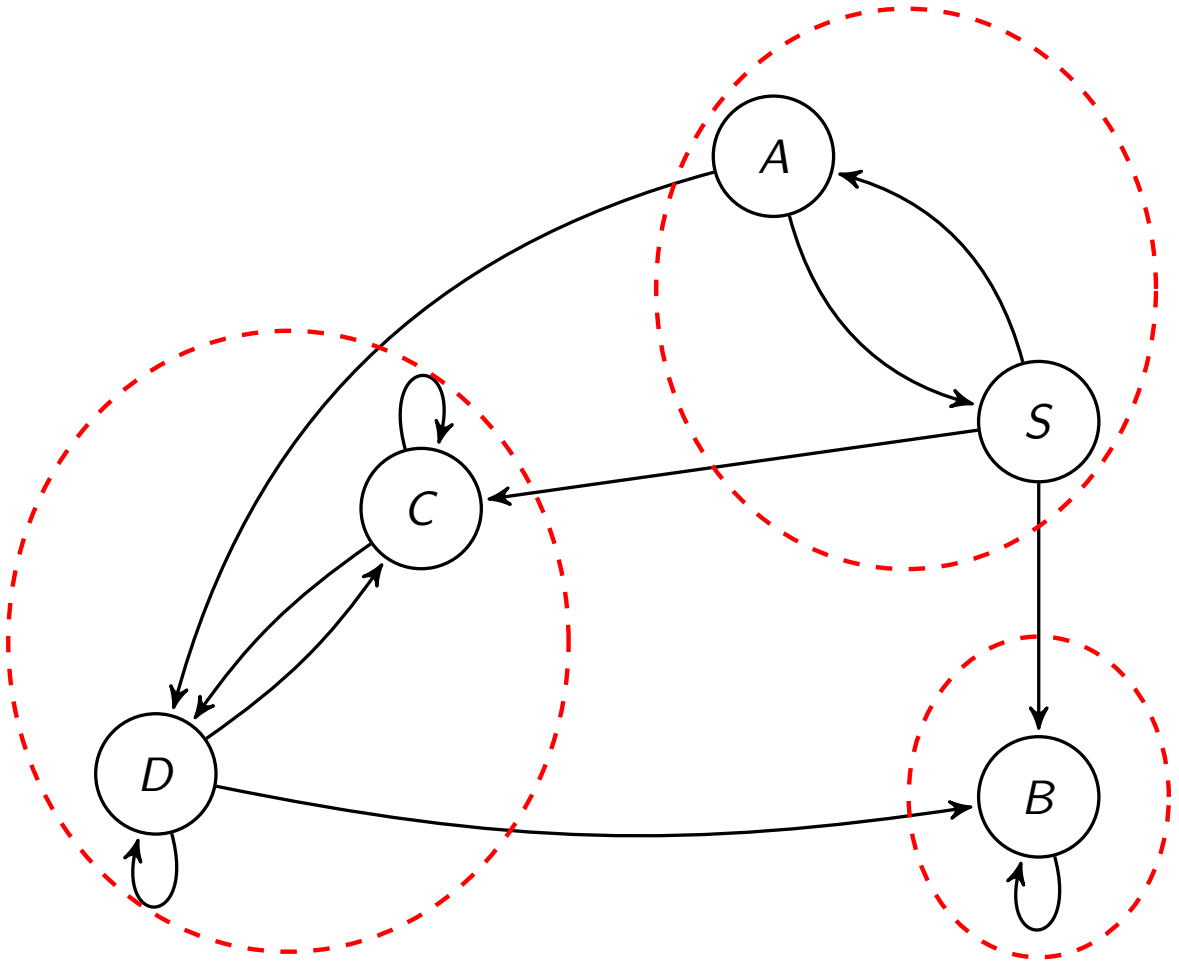

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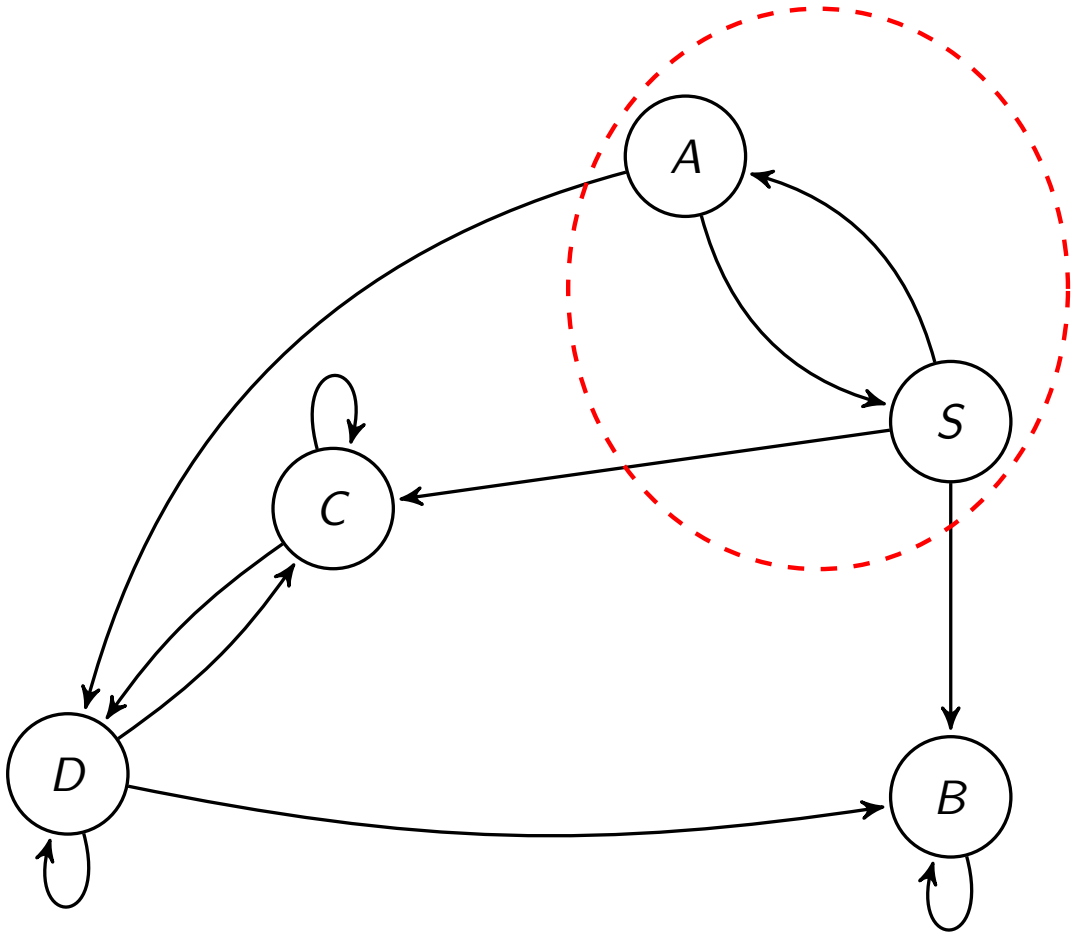

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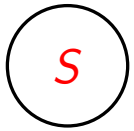

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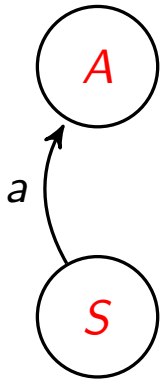

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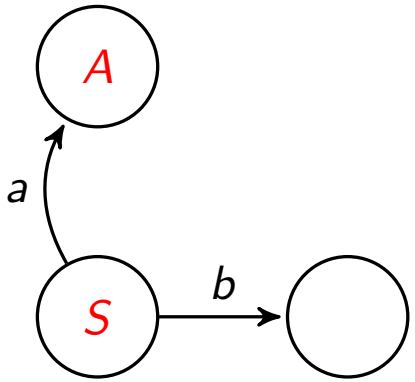

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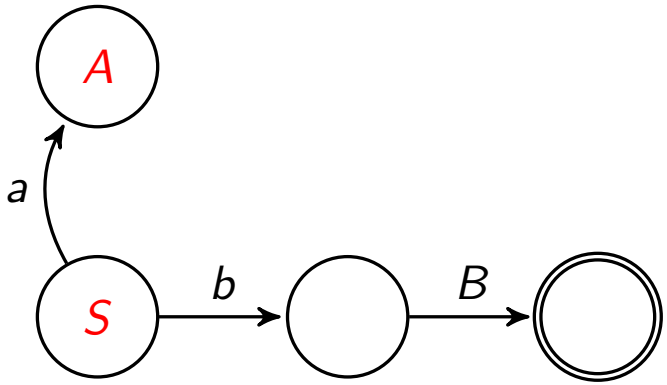

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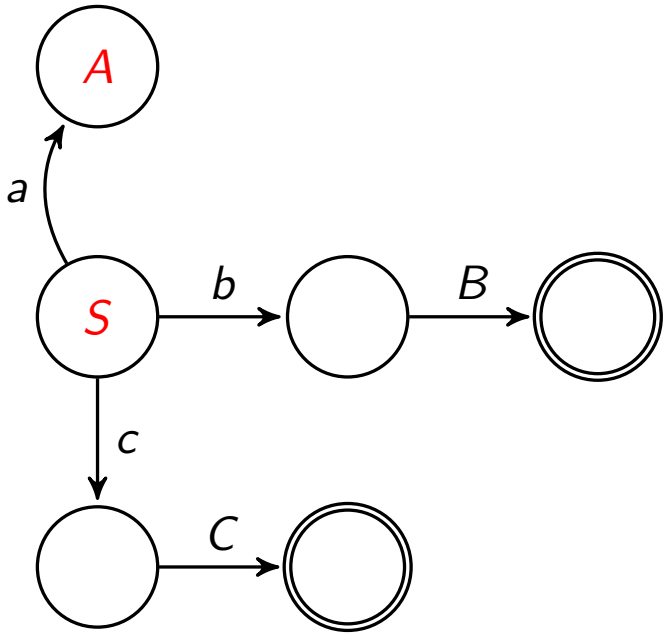

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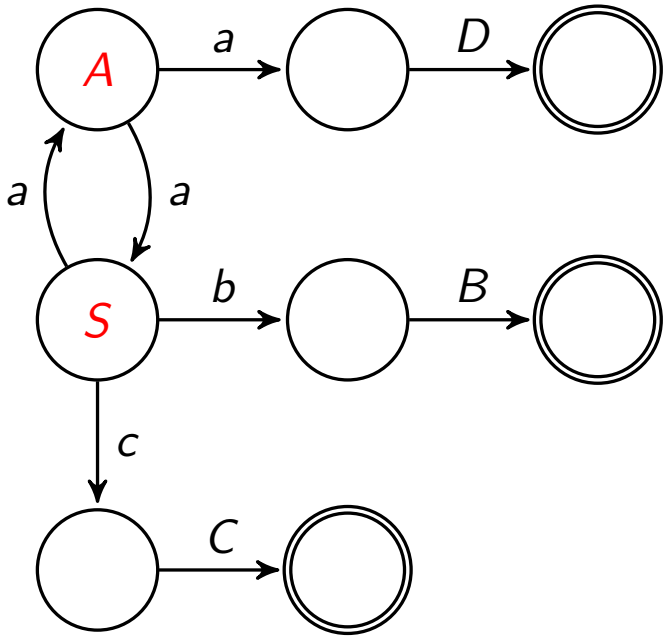

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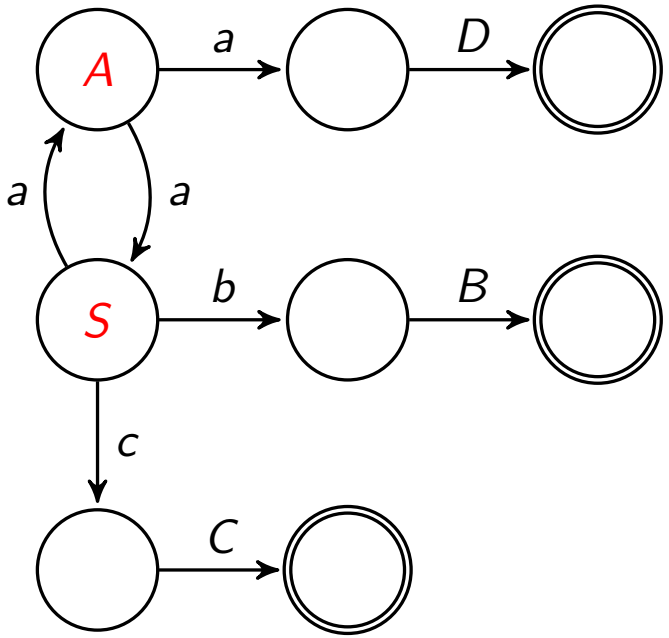

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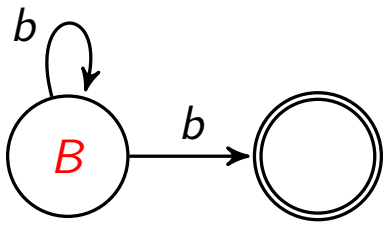

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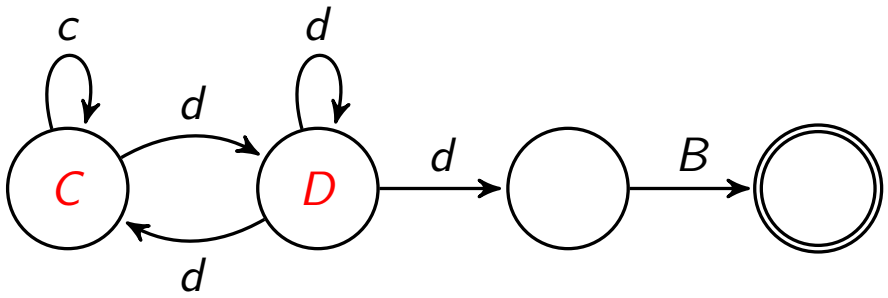

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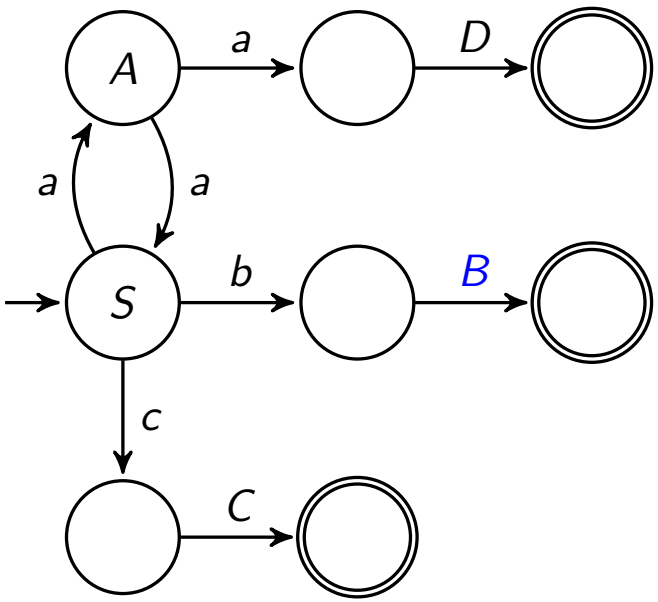

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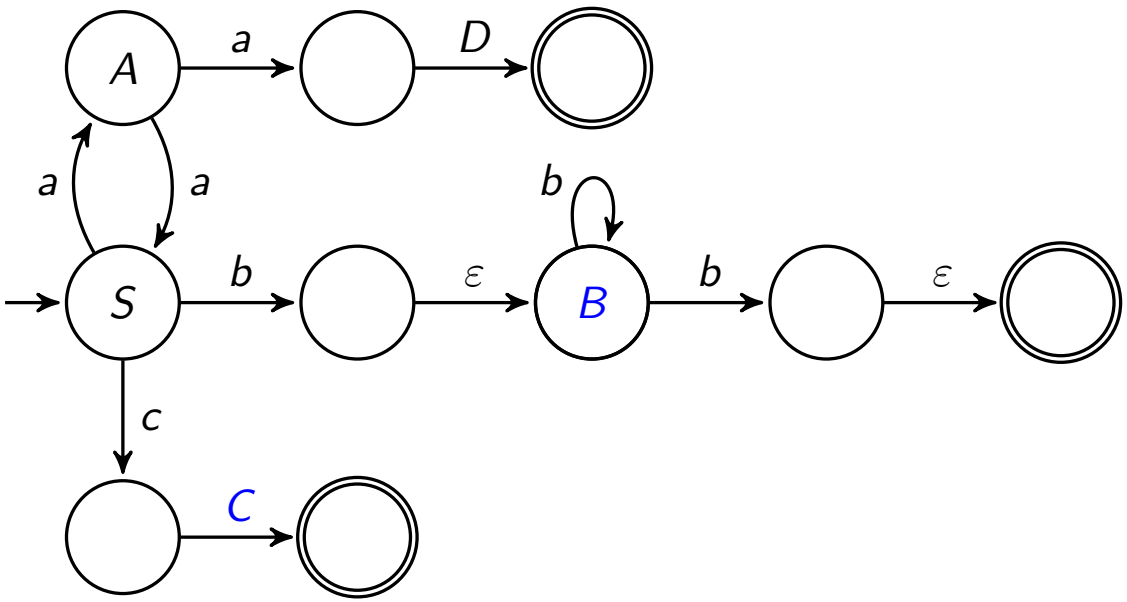

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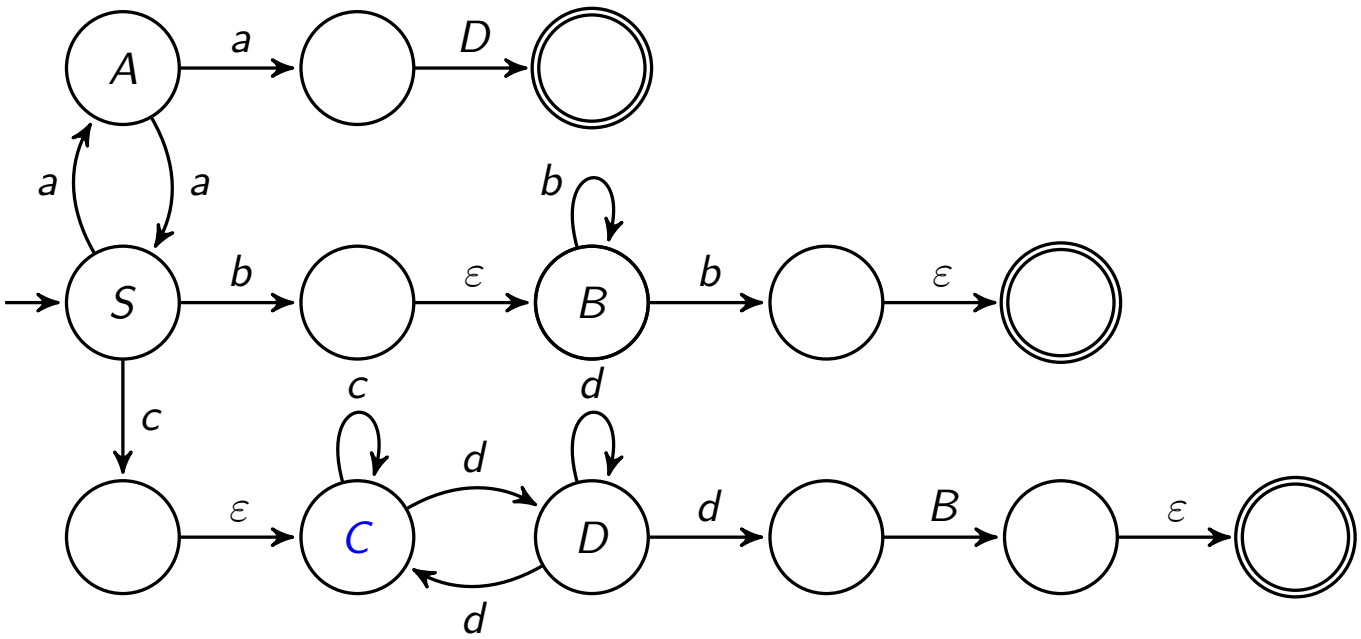

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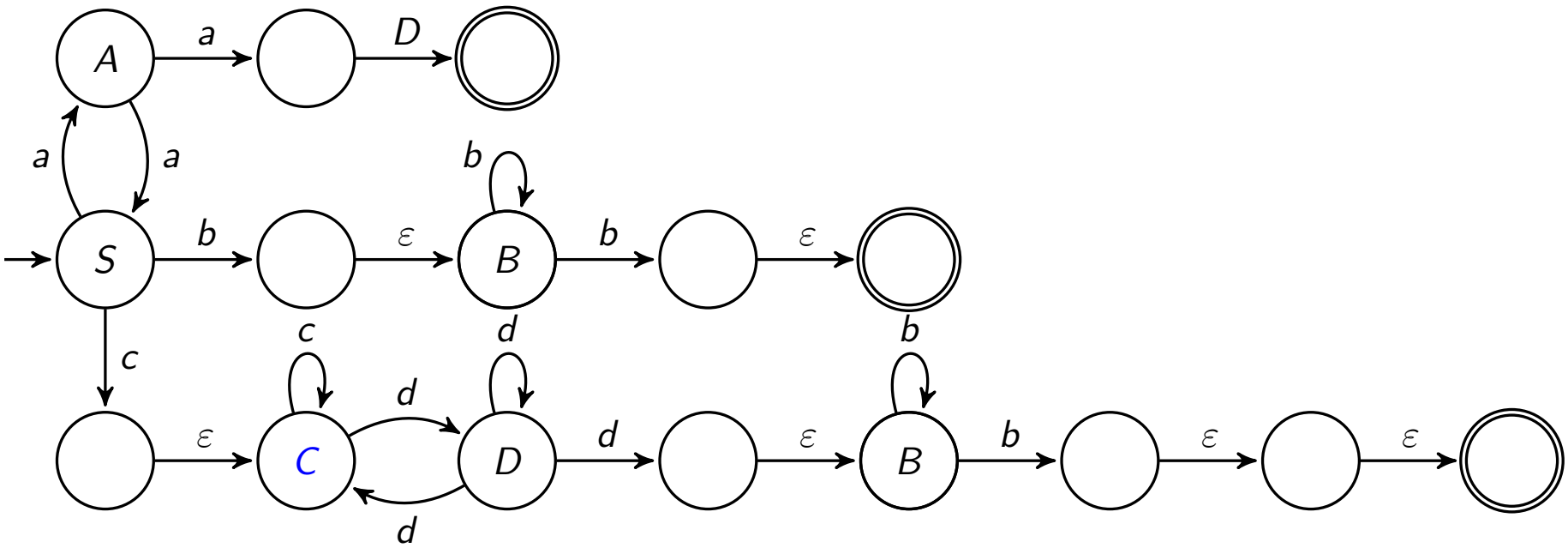

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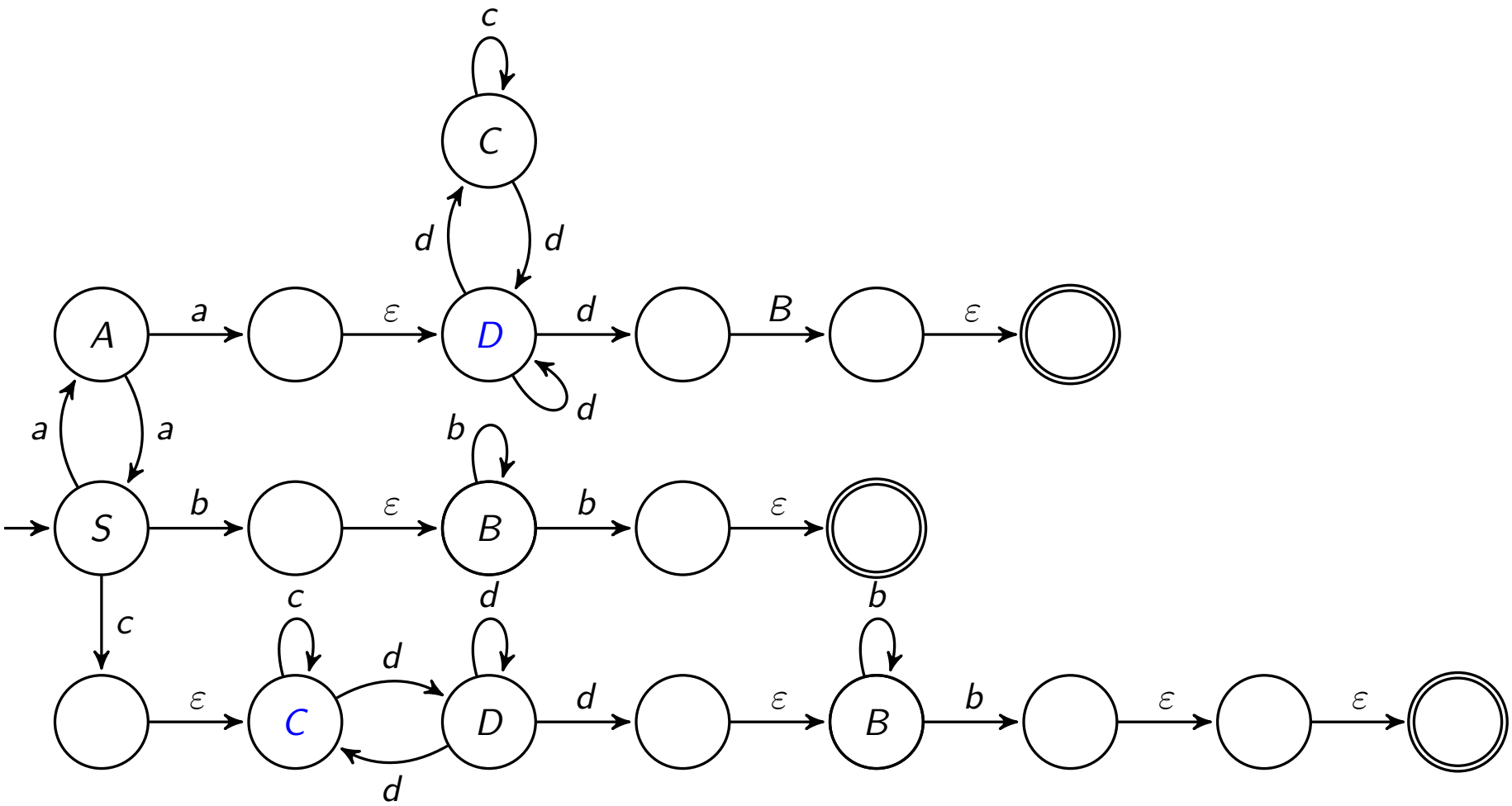

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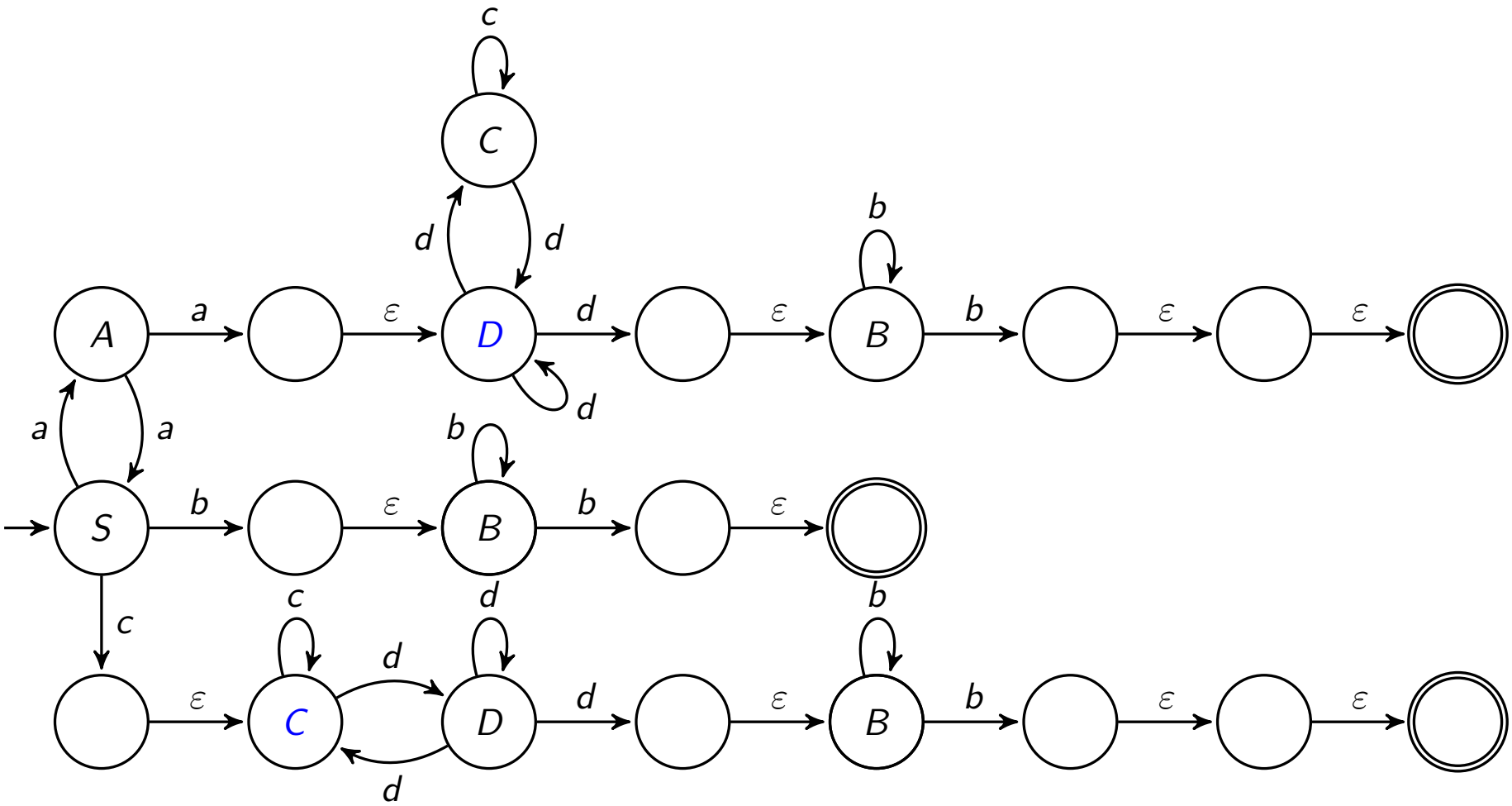

Non-Self-Embedding Grammars to Automata

Lower Bound for the Conversion

[Anselmo et al., 2002]

The size of the automaton accepting a regular language L is at least exponential with respect to a NSE grammar generating L.

$$A_n \to A_{n-1}A_{n-1}$$
$$A_{n-1} \to A_{n-2}A_{n-2}$$
$$\vdots$$
$$A_2 \to A_1 A_1$$
$$A_1 \to A_0 A_0$$
$$A_0 \to a$$

$$L = \{a^{2^n}\}$$

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Upper Bound for the Conversion

- Grammar G of size $s = s_1 + s_2 + \ldots + s_n$ (Step I)
- Each NFA M_i (Step II) has size $O(s_i)$
- The substitution of the automaton M_i in M_j (Step III) has size $O(s_i s_j)$

$$s_1 \cdots s_n = 2^{\log(s_1 \cdots s_n)} = 2^{\log s_1 + \cdots + \log s_n}$$

- This leads to a 2^{2^s}-state DFA

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Tightness: the Witness Language

$$L_n = a_1 \quad a_2 \quad \cdots \quad a_n \quad \cdots \; a_{j+1} \; a_{j+2} \; \cdots \; a_{j+n} \; \cdots \quad a_{\ldots} \quad \cdots a_{kn-1} \; a_{kn}$$

$$L_n = \{x_1 \cdots x_k \quad | \quad x_i \in \{a,b\}^n, \; k > 1, \; i = 1, \ldots, k, \; \exists j, 1 \leq j < k, \text{ s.t. } x_j = x_k^R\}$$

The grammar generating L_n has size $O(n)$

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x_k is equal to the reversal of some x_j

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- With an NFA:
  1. Guess which is the block x_j
  2. Store x_j
  3. Guess which is the last block
  4. Verify $x_k^R = x_j$
- $O(2^n)$ states
- By a fooling set argument, any equivalent DFA requires 2^{2^n} states

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Recognizing L_n

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The double exponential bound NSE $\rightarrow$ DFA is tight

Quasi-NSE Grammars: Definition

- Unary CFGs only generate regular languages

  [Ginsburg and Rice, 1962]

- Context-Free Grammar $G = (V, \Sigma, P, S)$
- $A \stackrel{\star}{\Longrightarrow} \alpha A \beta$
- α, β are not empty iff A only generates a unary language
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Bounds for unary CFGs grammars [Pighizzini et al., 2002]

- CFG with size s
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Back to the Witness Language

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- Each block could have 2^n different configurations
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Study the letter-bounded case

The Letter-Bounded Case

- G is a qNSE of size s
- $L(G) \subseteq a_1^* a_2^* \cdots a_m^*$, for $\Sigma = \{a_1, a_2, \ldots, a_m\}$
- The equivalent NFA has $2^{O(s)}$
- [Herrmann et al., 2016] $\mathrm{NFA} \to \mathrm{DFA}$ bounded case: subexponential but still superpolynomial cost
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We can do better!

- Each "unary SCC" of size s' can be replaced by a $2^{O(s'^2)}$-state DFA.

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letter-bounded $\mathrm{qNSE} \to \mathrm{NFA}/\mathrm{DFA}$ exponential

Conclusion

	NFA	DFA
(q)NSE	exponential	double-exponential
l.b. qNSE	exponential	exponential

Thanks for your attention

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