Two-Sided Strictly Locally Testable Languages

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Overview

- Two-Sided Strictly Locally Testable Languages
- Expressive Capacity
- Learnability in the Limit
- Decidability Problems
- Closure Properties

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Standard Languages

Standard language: $L \subseteq \Sigma^*$ can be described by three sets $A, B \subseteq \Sigma$ and $C \subseteq \Sigma^2$ such that

$$L = (A\Sigma^* \cap \Sigma^* B) \setminus \Sigma^* F \Sigma^*$$

where $F = \Sigma^2 \setminus C$ is the set of forbidden factors.

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Theorem **[Chomsky and Schützenberger 1963]**

Every regular language is the letter-to-letter homomorphic image of a standard language.

Two-Sided Strictly Locally Testable Languages

Strictly Locally Testable Languages

Strictly k-testable language: $L \subseteq \Sigma^*$ can be described by three sets $A, B, C \subseteq \Sigma^k$ such that, if $|w| \geq k$ then $w \in L$ if and only if

its prefix of length k is in A, its suffix of length k is in B,
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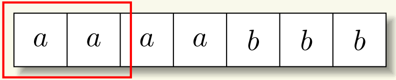
Example

The language $L = \{ a^m b^n \mid m, n \geq 1 \}$ is strictly 2-testable:

Set $A = \{aa, ab\}$, $B = \{ab, bb\}$, and $C = \{aa, ab, bb\}$.



$\in A$ ✓

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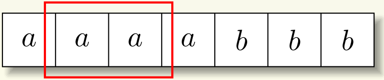

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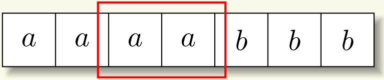

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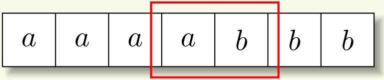

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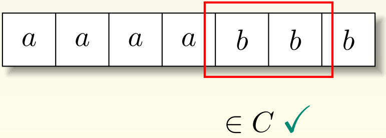

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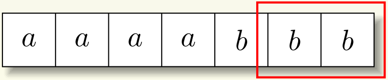
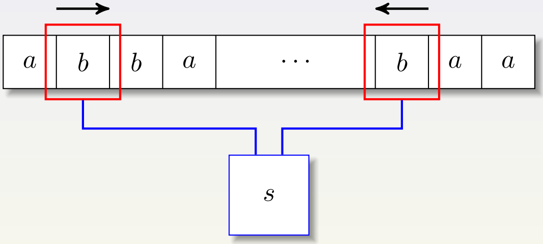


$\in B$ ✓

Two-Sided Strictly Locally Testable Languages

Double-Head Automata



A. Rosenberg 1967: Double-head finite automata characterize the linear context-free languages.

B. Nagy 2015: Double-head pushdown automata accept linguistically important languages and describe a family of mildly context-sensitive languages.

Two-Sided Strictly Locally Testable Languages

Idea: Two windows of some size k move across the input, one from left to right and the other from right to left.

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But: The families of strictly k-testable languages are closed under intersection.

Therefore: The windows move simultaneously and the factors scanned have to be in relation.

Two-sided strictly k-testable language: $L \subseteq \Sigma^*$ is given through a strictly k-testable language H and a binary relation $R \subseteq \Sigma^k \times \Sigma^k$.
If $|w| \geq k$ then $w \in L$ if and only if $w \in H$ and, for all indices $i \in \{1, 2, \ldots, |w| - k + 1\}$, $(w[i, k], w[|w| + 2 - k - i, k]) \in R$.

Two-Sided Strictly Locally Testable Languages

Example

There is a two-sided strictly 1-testable language L such that $L \cap a^*b^* = \{\, a^n b^n \mid n \geq 1 \,\}$:

Set $A = B = C = \{a, b\}$ and $R = \{(a, b), (b, a)\}$.

a	a	a	b	b	b

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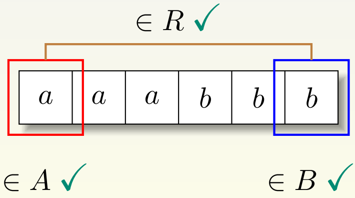

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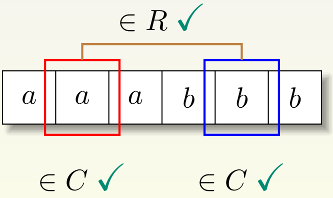

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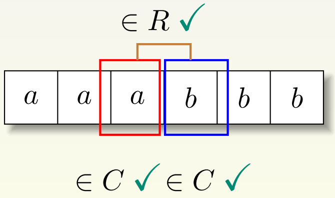

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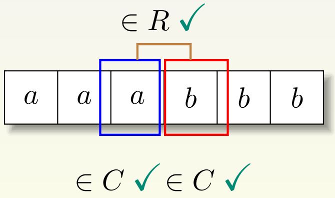

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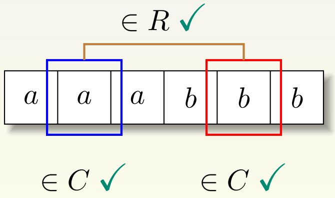

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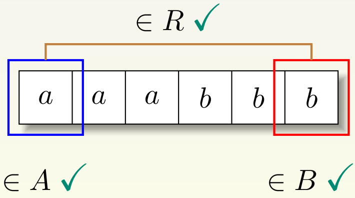
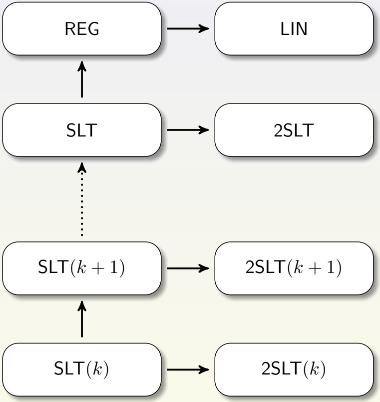


Expressive Capacity

Expressive Capacity

- SLT $\subset$ REG
- SLT$(k) \subset$ SLT$(k+1)$
- SLT$(k) \subset$ 2SLT(k)



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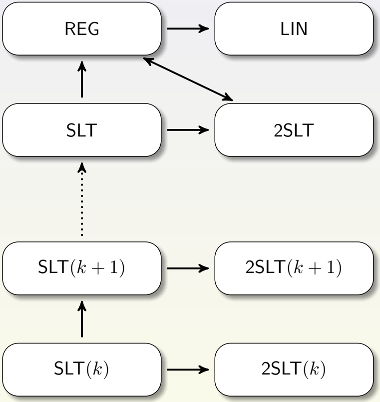

Theorem

The classes 2SLT and 2SLT(k), for $k \geq 1$, are incomparable to the classes REG, DLIN, DCFL, and CRL.

Expressive Capacity

Lemma

For all $k \geq 1$,
$\{ab^{k+1}\} \in \mathsf{SLT}(k+1) \setminus \mathsf{2SLT}(k)$.

Theorem

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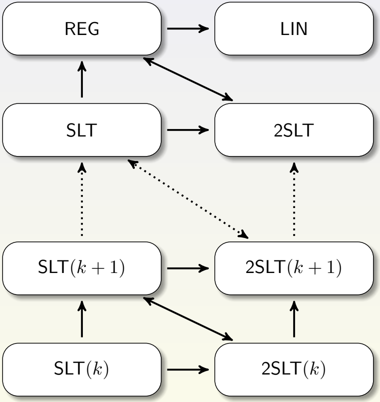

Expressive Capacity

Theorem

$2SLT \subset LIN$.

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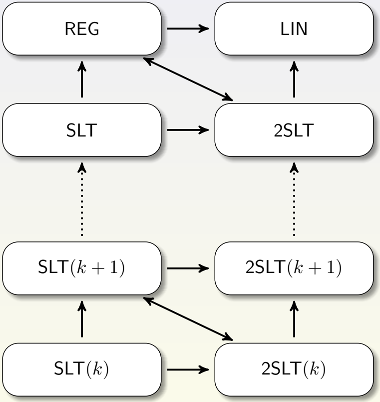

Expressive Capacity

- A unary language belongs to $\mathsf{SLT}(k)$ if and only if it belongs to $\mathsf{2SLT}(k)$.

Question: Do $\mathsf{SLT}(k)$ and $\mathsf{2SLT}(k)$ coincide for regular languages?

Expressive Capacity

- A unary language belongs to SLT(k) if and only if it belongs to 2SLT(k).

Question: Do SLT(k) and 2SLT(k) coincide for regular languages?

Answer: No

Theorem

There is a language belonging to $(\text{2SLT}(1) \cap \text{REG}) \setminus \text{SLT}$.

Witness language: $L_r = \{\, ac^n b, bc^n a \mid n \geq 0 \,\}$.

Learnability in the Limit

A **positive presentation** of a language L is an infinite sequence $\{w_j\}_{j=1}^{\infty}$ of words from L such that every $w \in L$ occurs at least once in the sequence.

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Identifying in the limit from positive data: There must exist an algorithm $\mathcal{A}$ that returns a conjecture automaton (grammar) M_j for any input $\{w_1, w_2, \ldots, w_j\}$ such that, for any positive presentation of L, there is a $j_0 \geq 1$ with $L(M_j) = L(M_{j+1}) = L$, for all $j \geq j_0$.

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Learnable in the limit from positive data: A class of languages $\mathscr{L}$ is learnable in the limit from positive data if there exists an algorithm $\mathcal{A}$ that, for each $L \in \mathscr{L}$, identifies L in the limit from positive data.

Learnability in the Limit

Theorem **[Gold 1967]**

Any class of languages including all finite languages and at least one infinite language is not learnable in the limit from positive data only.

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Theorem **[Yokomori, Kobayashi 1998]**

For any $k \geq 1$, the family of strictly k-testable languages is learnable in the limit from positive data.

Learnability in the Limit

Learning Algorithm:

Input: an integer $k \geq 1$ and a positive presentation $\{w_j\}_{j=1}^{\infty}$ of a two-sided strictly k-testable language L.

Output: a sequence of linear grammars G_j generating two-sided strictly k-testable languages such that

(i) for each $j \geq 0$, $L(G_j) \subseteq L(G_{j+1}) \subseteq L$, and

(ii) there exists $j_0 \geq 1$ such that, for each $j \geq j_0$, $G_j = G_{j+1}$, and $L(G_j) = L$.

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Remark

The families SLT and 2SLT are not learnable in the limit from positive data.

Decidability Problems

Basic properties

- ➜ Finiteness, infiniteness, and emptiness are decidable in polynomial time for two-sided strictly locally testable languages.

Decidability Problems

Universality

- ➜ Universality is undecidable for linear context-free languages.

Theorem

Universality is decidable in linear time for two-sided strictly locally testable languages.

Decidability Problems

Universality – Regular Inclusion Problem

INSTANCE: A regular language S and
a two-sided strictly k-testable language L.

QUESTION: Is S contained in L?

Decidability Problems

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The regular inclusion problem is decidable for two-sided strictly locally testable languages.

Decidability Problems

Regular Equality Problem

INSTANCE: A regular language S and
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QUESTION: Is $S = L$?

- Essentially, this question reduces to the questions of whether the inclusion $L \subseteq S$ is true.
- $L \subseteq S$ if and only if $L \cap \overline{S} = \emptyset$.

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Theorem

The regular equality problem is decidable for two-sided strictly locally testable languages.

Decidability Problems

General Inclusion and Equivalence

Theorem

For any $k \geq 1$, the inclusion and equivalence problems for two-sided strictly k-testable languages are decidable in polynomial time.

Decidability Problems

Theorem

It is decidable whether a given two-sided strictly k-testable language is strictly k-testable.

Open Problem

The decidability status of the regularity problem for two-sided strictly k-testable languages is open.

Closure Properties

Language class	c	$\cup$	$\cap$	R	$\cdot$	$*$	$h_{\text{len.pres.}}$	$h^{-1}_{\text{len.pres.}}$	h^{-1}_{λ}
2SLT	✗	✗	✓	✓	✗	✗	✗	✓	✗
2SLT(k)	✗	✗	✓	✓	✗	✗	✗	✓	✗