Inhabitation Machines: Determinism and Principality

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  - backward inhabitation search; context-free grammars; formula-tree method; game semantics; etc.
  - Schubert et al. (2015) use techniques from automata theory reproving PSPACE-completeness of the emptiness problem;

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- the resulting inhabitation machines work deterministically;
- we extend the machines to address principal inhabitation.

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Examples: $\lambda xy.x(\lambda z.y)y$, $\lambda x.x$, ...

Simply Typed Lambda Calculus

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The set of simple types $\mathcal{T}$

- if a is a type variable, then $a \in \mathcal{T}$;
- $\alpha, \beta \in \mathcal{T} \quad \Rightarrow \quad (\alpha \to \beta) \in \mathcal{T}$;
- associativity to the right:
  $\alpha_1 \to \alpha_2 \to \cdots \to \alpha_n$ instead of $(\alpha_1 \to (\alpha_2 \to (\cdots \to \alpha_n)))$.

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Every type α can be uniquely written as $\alpha = \alpha_1 \to \ldots \to \alpha_n \to a$, where a is a type-variable and $n \geq 0$.

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Example $\alpha = ((o \to o) \to o \to o) \to o \to o$

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$\lambda x^{\tau_1} y^o . x^{\tau_1} (\lambda z^o . z^o)(x^{\tau_1} (\lambda u^o . y^o) y^o)$

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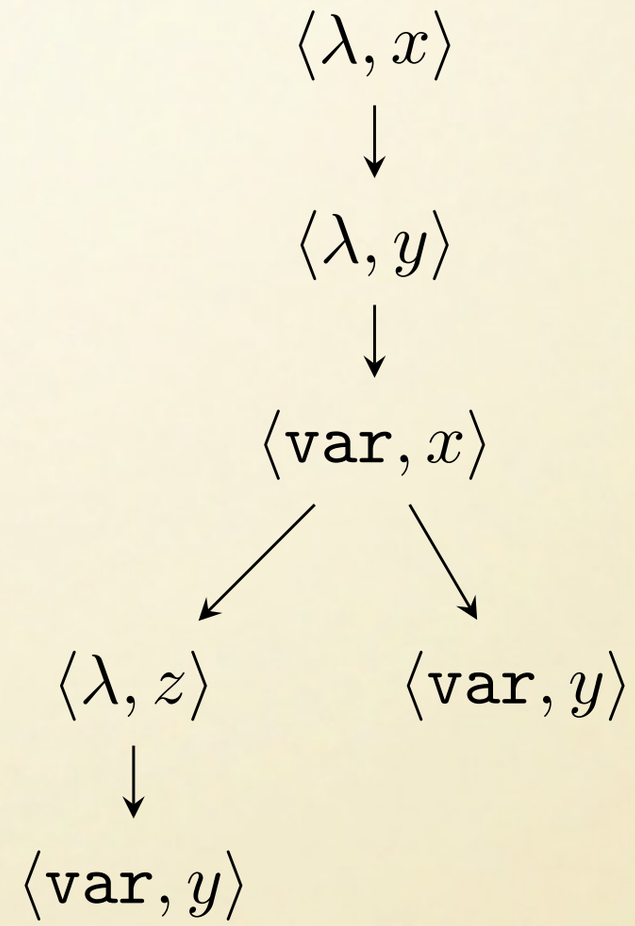

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$$a, q, \texttt{r} \rightsquigarrow q_1, \ldots, q_n, \texttt{r}'$$

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- the initial configuration is $\mathtt{c} = (\mathbf{t}, q_I, \mathtt{R}_\varnothing)$

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A non-empty sequence $\mathtt{c}_1, \mathtt{c}_2, \ldots, \mathtt{c}_n$ with $\mathtt{c}_1 = (\mathbf{t}_1, q_i, \mathtt{R}_1)$ transitions to $\mathtt{c}^1, \ldots, \mathtt{c}^k, \mathtt{c}_2, \ldots, \mathtt{c}_n$ if one of the following applies:

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- $\mathbf{t}_1$ has root $\langle \lambda, x \rangle$ and a single subtree $\mathbf{t}'$, there is some rule $\lambda, q_i, \varnothing \rightsquigarrow q^1, \mathtt{r}$ in δ, $k = 1$, and $\mathtt{c}^1 = (\mathbf{t}', q^1, \mathtt{R}')$, where $\mathtt{R}'$ is obtained from $\mathtt{R}_1$ by updating register $\mathtt{r}$ with the addition of variable x

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- the set of normal terms M such that $\mathcal{A}$ accepts $\mathbf{t}^M$ is denoted by $\mathcal{L}(\mathcal{A})$

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- $n \equiv_{\texttt{OccT}} m$, if and only if they correspond to the same subtype

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o	2	`var`
o	3	`var`
o	4	`var`
o	5	`var`
$o \to o$	6	$0 \to 1$
$o \to o$	7	$2 \to 3$
$o \to o$	8	$4 \to 5$
$(o \to o) \to o \to o$	9	$6 \to 7$
α	10	$9 \to 8$

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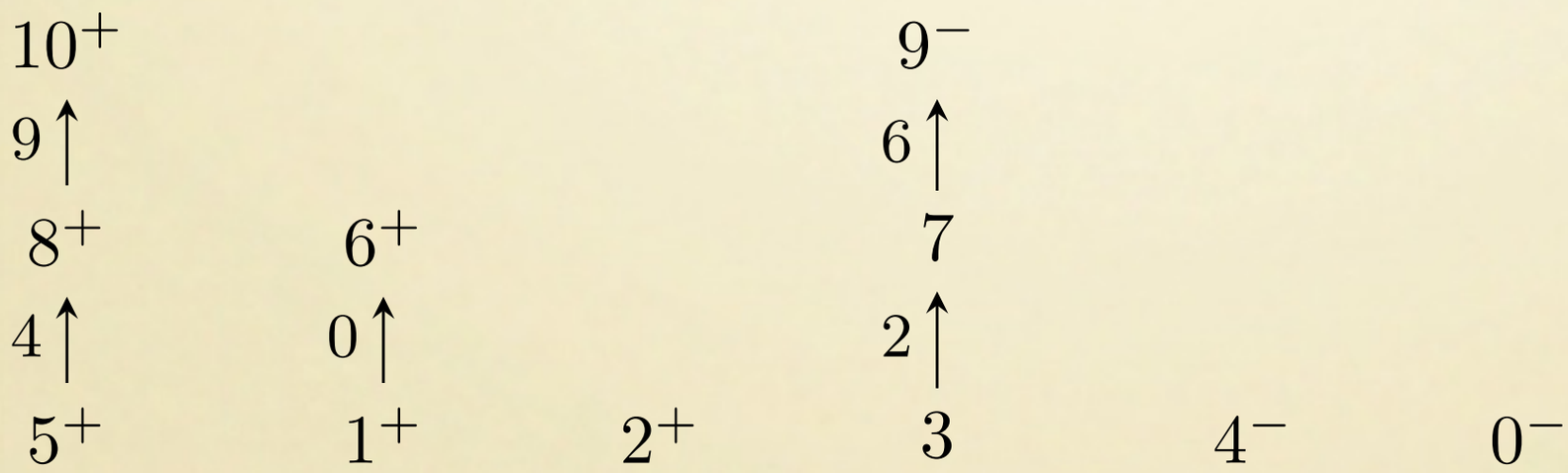

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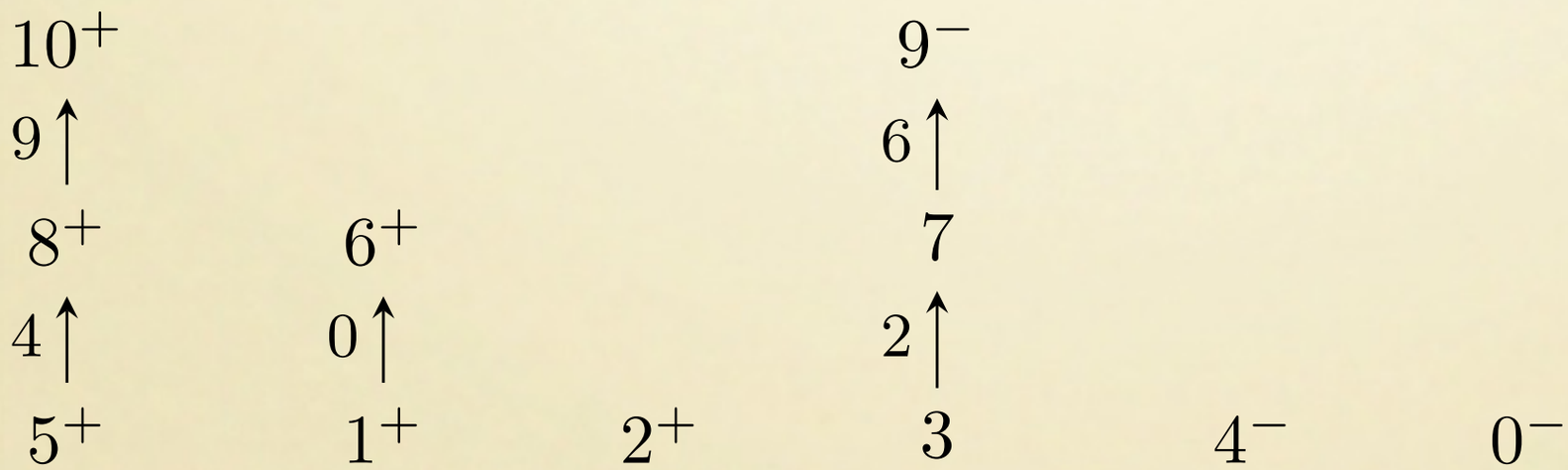
$$
\begin{array}{lcl}
10 & := & \lambda 9.8 \\
8 & := & \lambda 4.5 \mid 9\ 6 \\
6 & := & \lambda 0.1 \mid 9\ 6 \\
5 & := & 9\ 6\ 2 \mid 4 \mid 0 \\
2 & := & 9\ 6 \mid 4 \mid 0 \\
1 & := & 9\ 6\ 2 \mid 4 \mid 0
\end{array}
$$



Inhabitation

Inhabitation

$$
\begin{array}{lll}
10 & := & \lambda 9.8 \\
8 & := & \lambda 4.5 \mid 9\ 6 \\
6 & := & \lambda 0.1 \mid 9\ 6 \\
5 & := & 9\ 6\ 2 \mid 4 \mid 0 \\
2 & := & 9\ 6 \mid 4 \mid 0 \\
1 & := & 9\ 6\ 2 \mid 4 \mid 0
\end{array}
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$$A_\alpha = \langle \{\lambda, \mathtt{var}\}, \mathcal{V}, Q, q_{10}, \mathcal{R}, \delta \rangle$$

Inhabitation

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\lambda, q_{10}, \varnothing & \rightsquigarrow & q_8, \mathtt{r}_9 \\
\lambda, q_8, \varnothing & \rightsquigarrow & q_5, \mathtt{r}_4 \\
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\lambda, q_6, \varnothing & \rightsquigarrow & q_1, \mathtt{r}_0 \\
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\mathtt{var}, q_5, \mathtt{r}_9 & \rightsquigarrow & q_6, q_2, \varnothing \\
\mathtt{var}, q_5, \mathtt{r}_4 & \rightsquigarrow & \varnothing \\
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\mathtt{var}, q_2, \mathtt{r}_9 & \rightsquigarrow & q_6, q_2, \varnothing \\
\mathtt{var}, q_2, \mathtt{r}_4 & \rightsquigarrow & \varnothing \\
\mathtt{var}, q_2, \mathtt{r}_0 & \rightsquigarrow & \varnothing \\
\mathtt{var}, q_1, \mathtt{r}_9 & \rightsquigarrow & q_6, q_2, \varnothing \\
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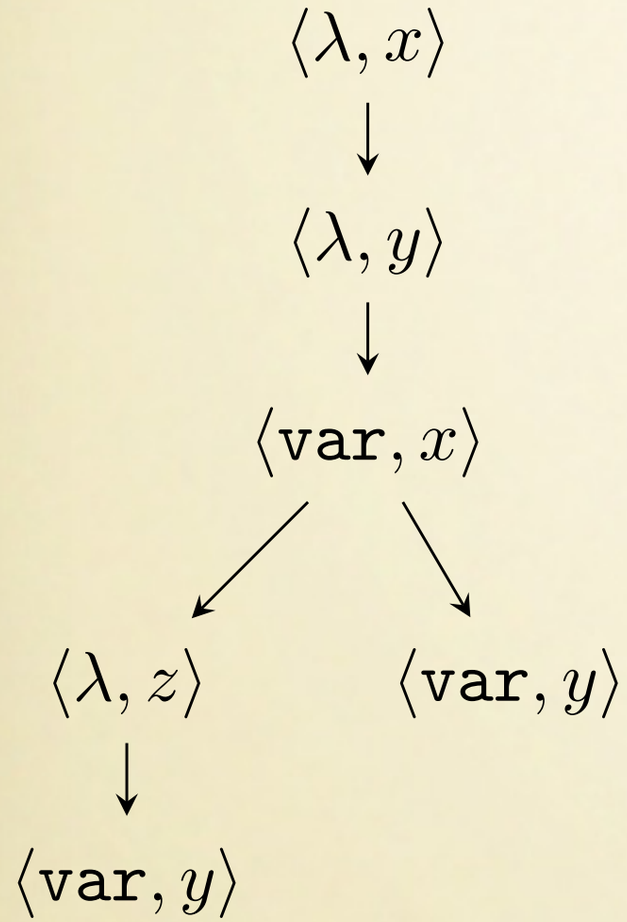
\mathtt{var}, q_1, \mathtt{r}_0 & \rightsquigarrow & \varnothing
\end{array}
$$

Inhabitation

Inhabitation



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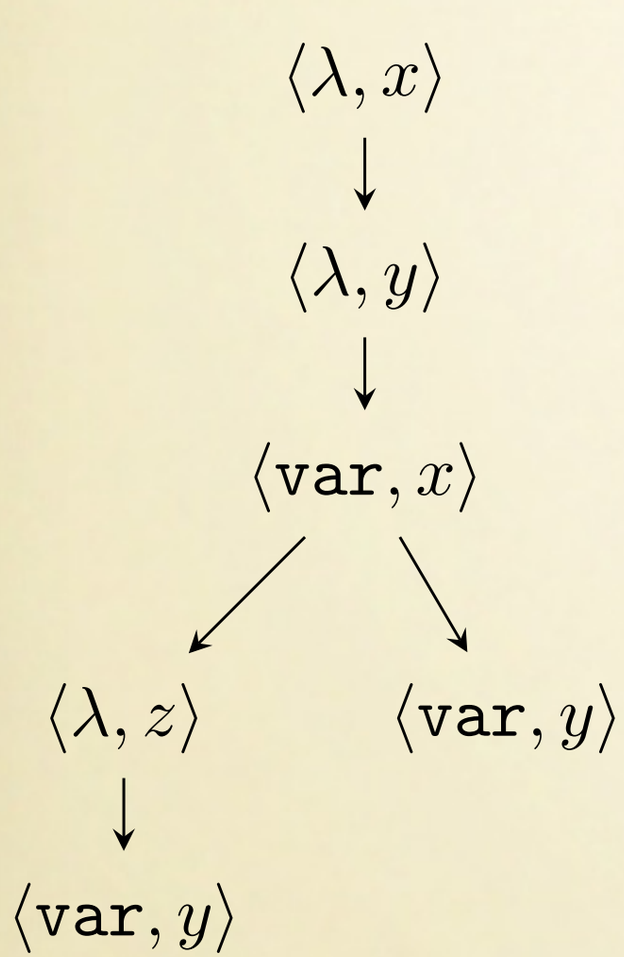
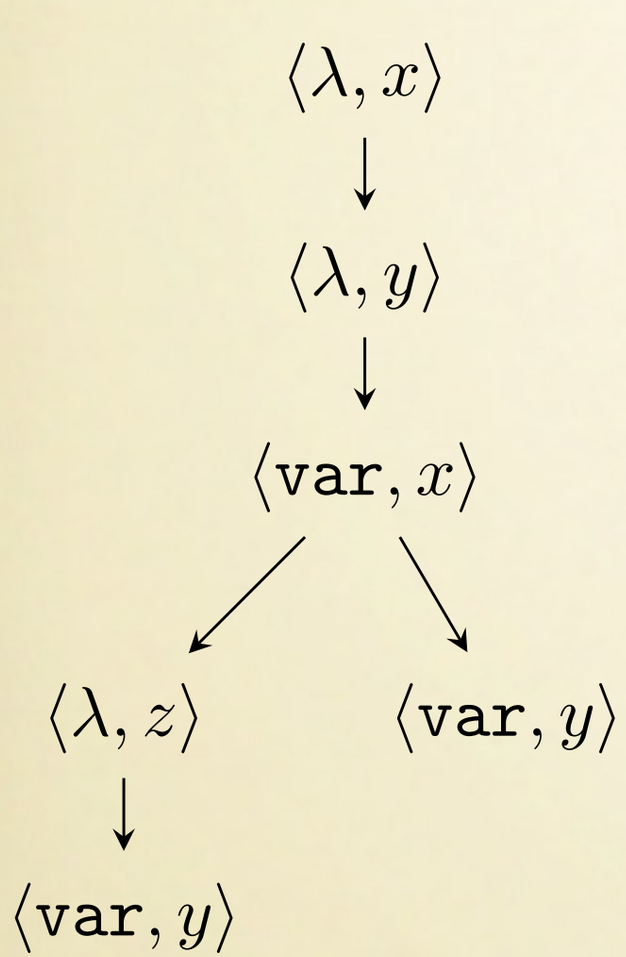


$(q_{10}, [\emptyset_9, \emptyset_4, \emptyset_0])$

Inhabitation



$$\lambda, q_{10}, \varnothing \rightsquigarrow q_8, \mathtt{r}_9$$

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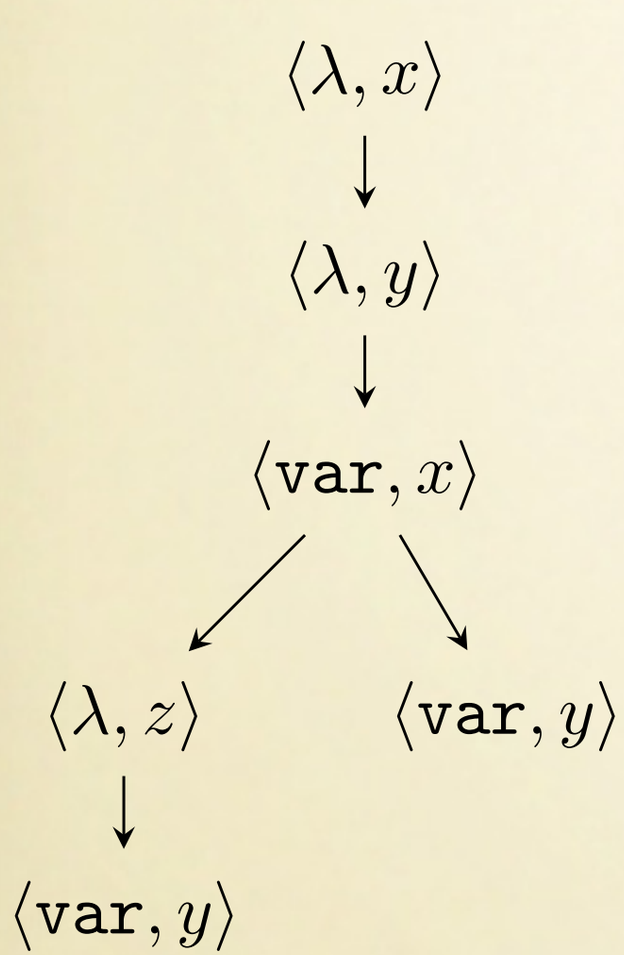
Inhabitation

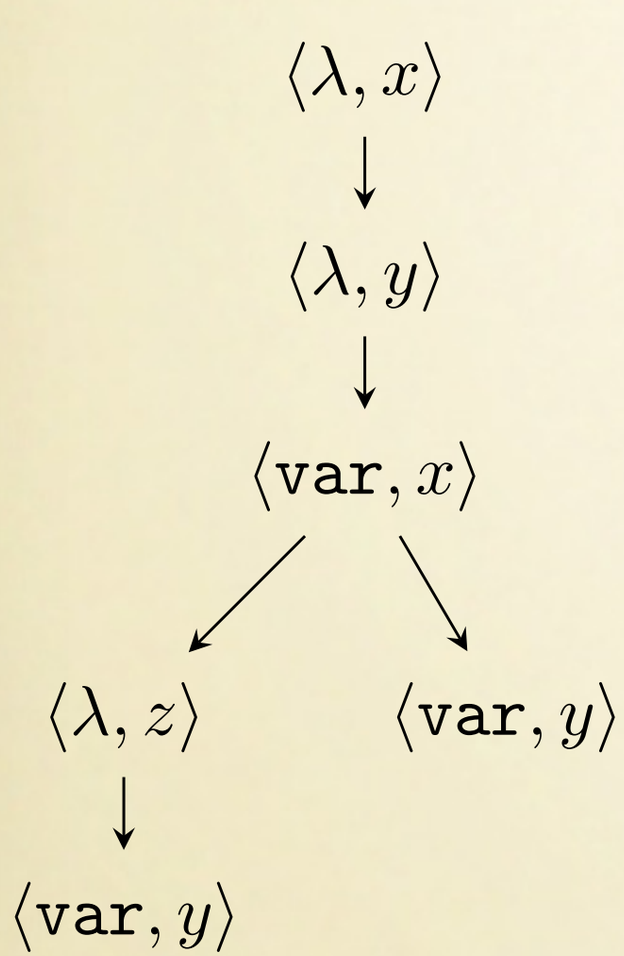


$$\lambda, q_{10}, \varnothing \rightsquigarrow q_8, \mathtt{r}_9$$

$$(q_{10}, [\emptyset_9, \emptyset_4, \emptyset_0])$$

$$(q_8, [\{x\}_9, \emptyset_4, \emptyset_0])$$

Inhabitation



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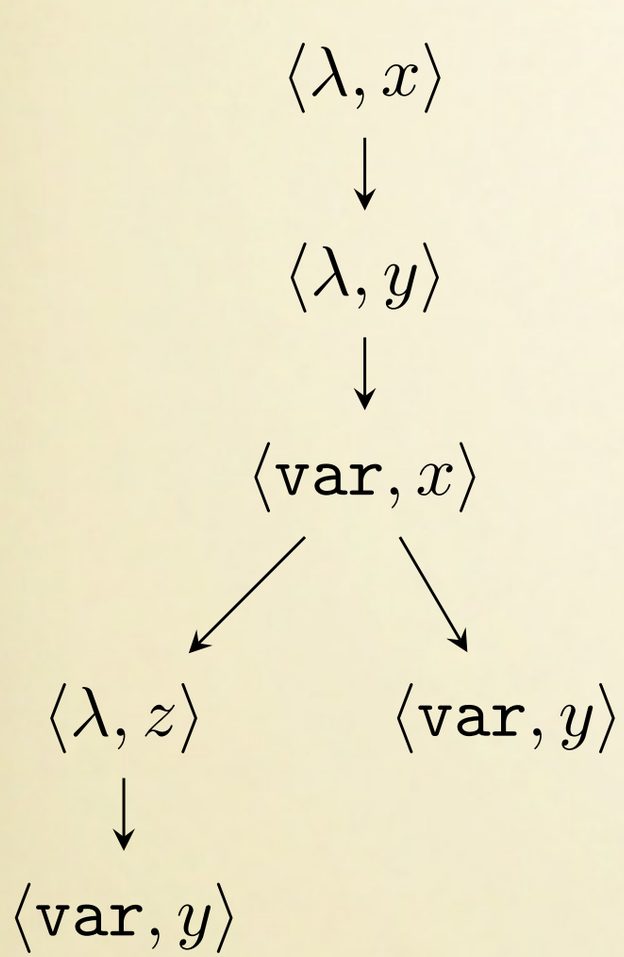

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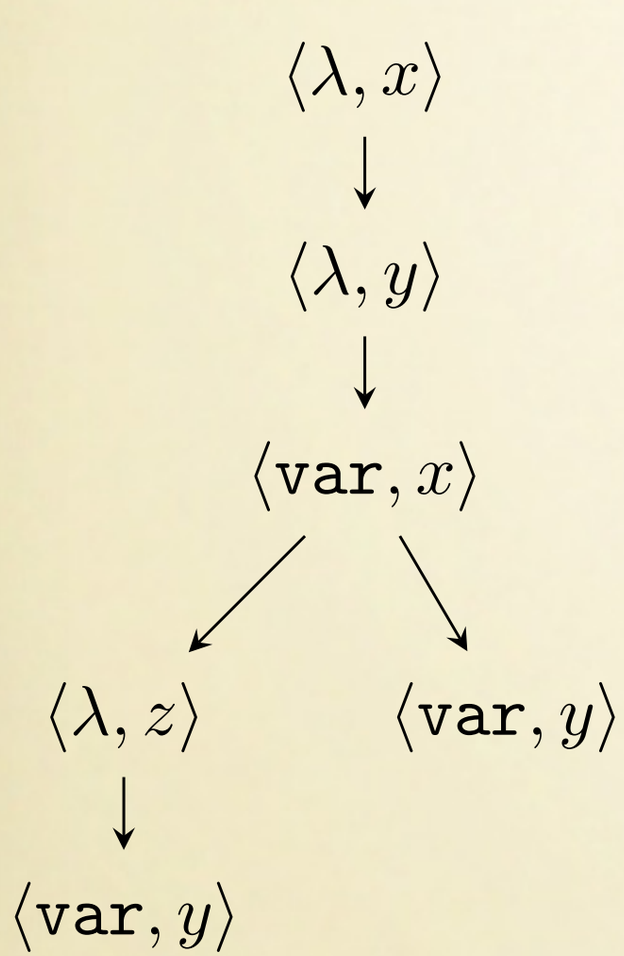
$$(q_5, [\{x\}_9, \{y\}_4, \emptyset_0])$$

Inhabitation



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$$\mathtt{var}, q_5, \mathtt{r}_9 \rightsquigarrow q_6, q_2, \varnothing$$

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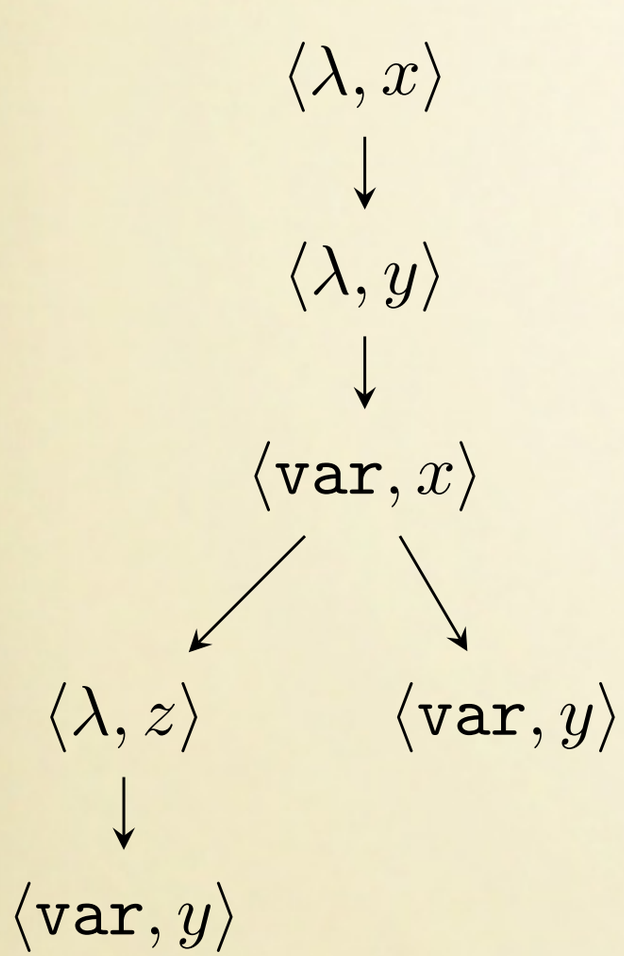

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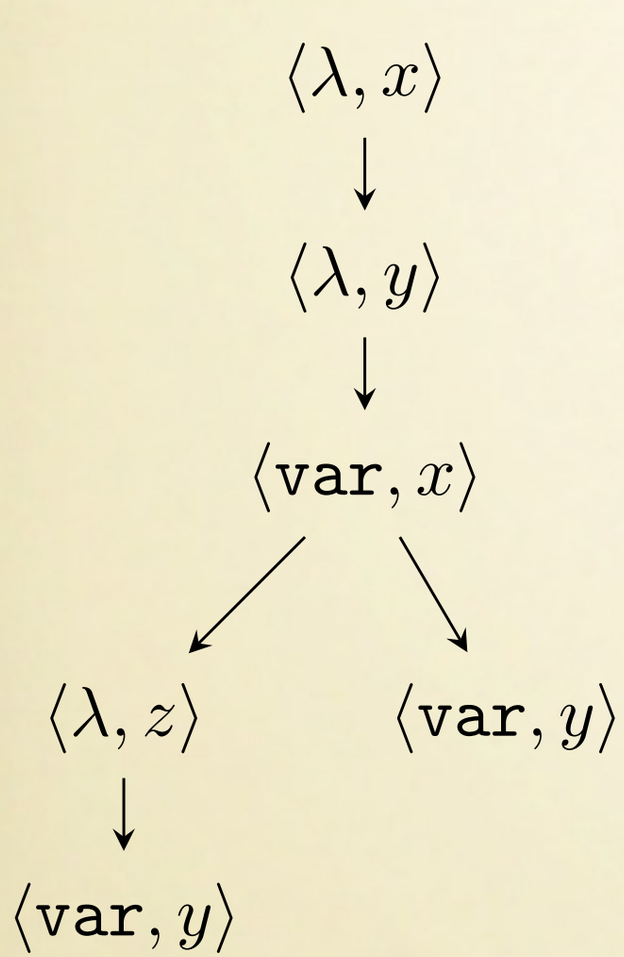
Inhabitation



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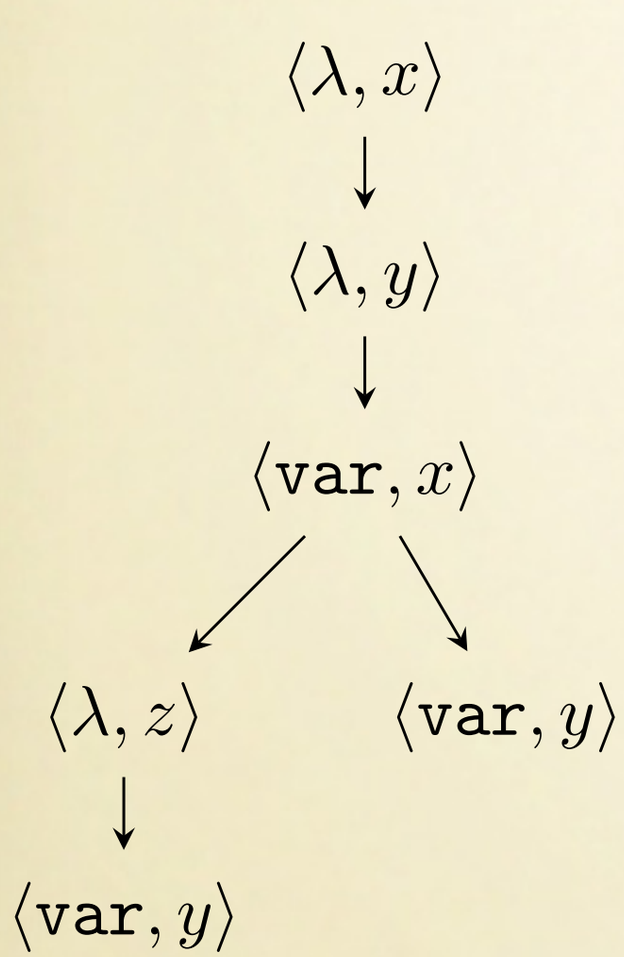

$\lambda, q_{10}, \varnothing \rightsquigarrow q_8, \mathtt{r}_9$

$(q_{10}, [\emptyset_9, \emptyset_4, \emptyset_0])$

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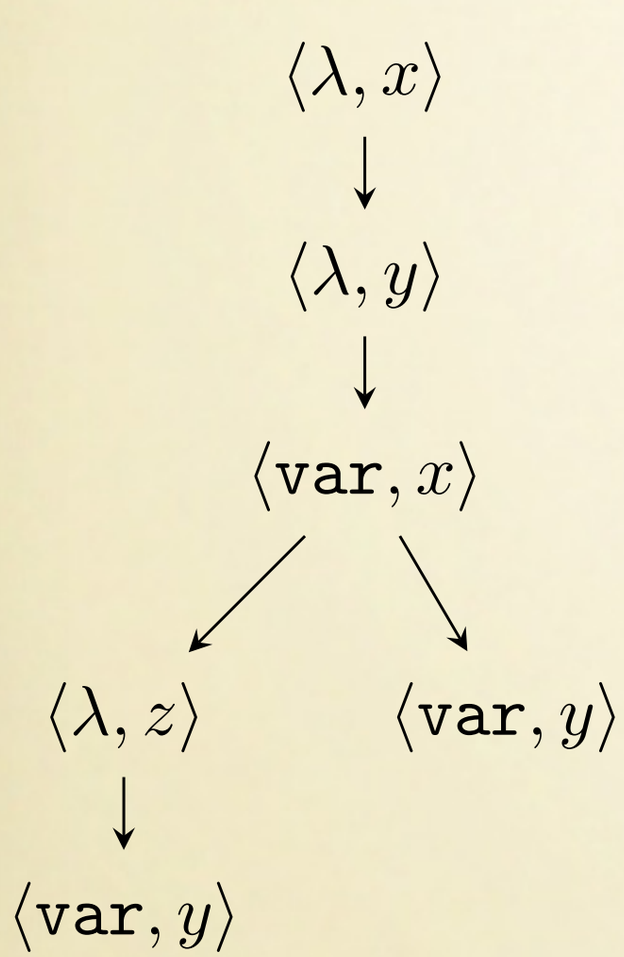
$\lambda, q_6, \varnothing \rightsquigarrow q_1, \mathtt{r}_0$

$(q_6, [\{x\}_9, \{y\}_4, \emptyset_0]), (q_2, [\{x\}_9, \{y\}_4, \emptyset_0])$

$(q_1, [\{x\}_9, \{y\}_4, \{z\}_0]), (q_2, [\{x\}_9, \{y\}_4, \emptyset_0])$

Inhabitation



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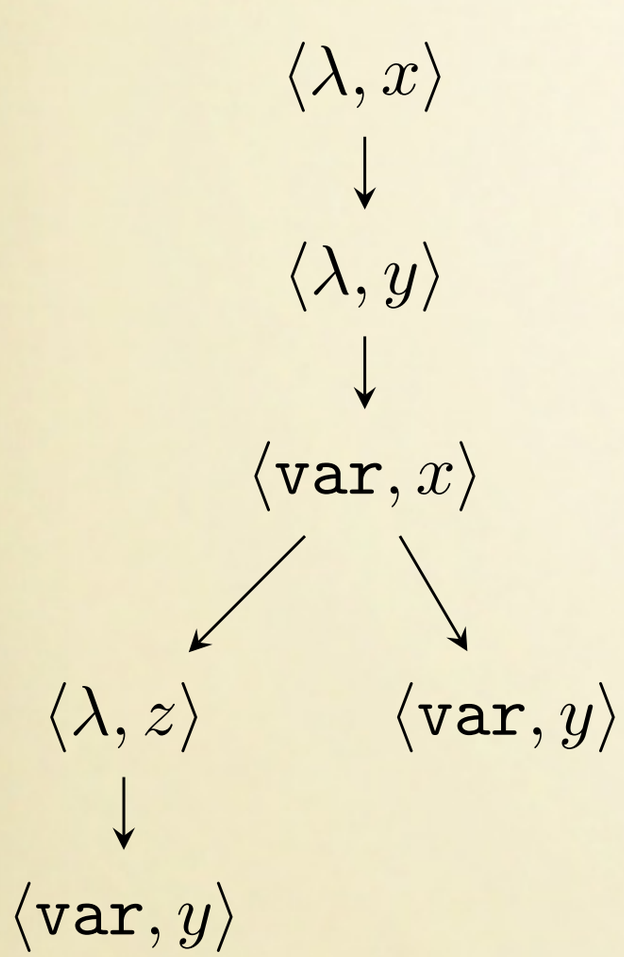

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$(q_6, [\{x\}_9, \{y\}_4, \emptyset_0]), (q_2, [\{x\}_9, \{y\}_4, \emptyset_0])$

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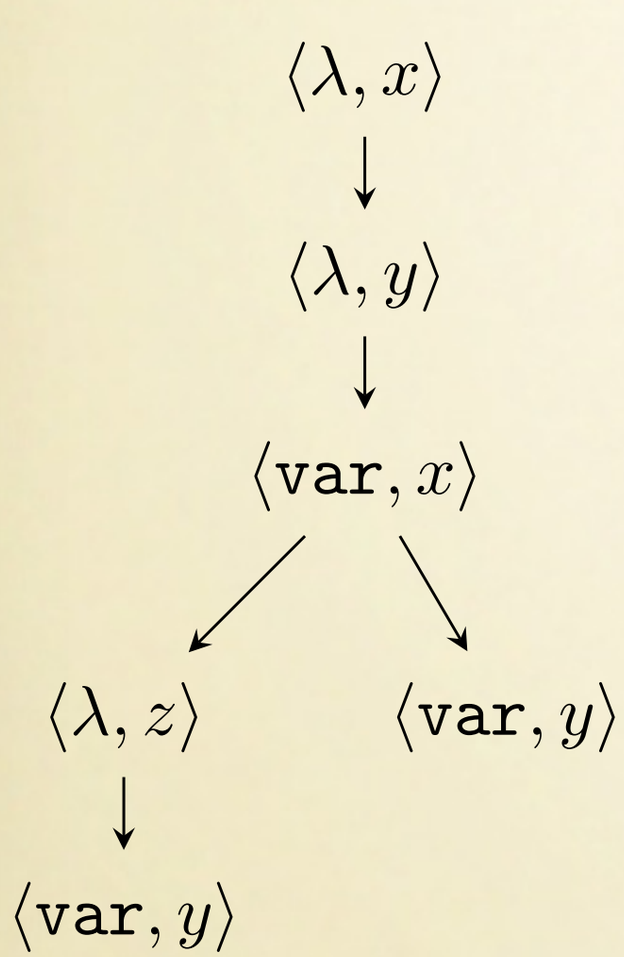
Inhabitation



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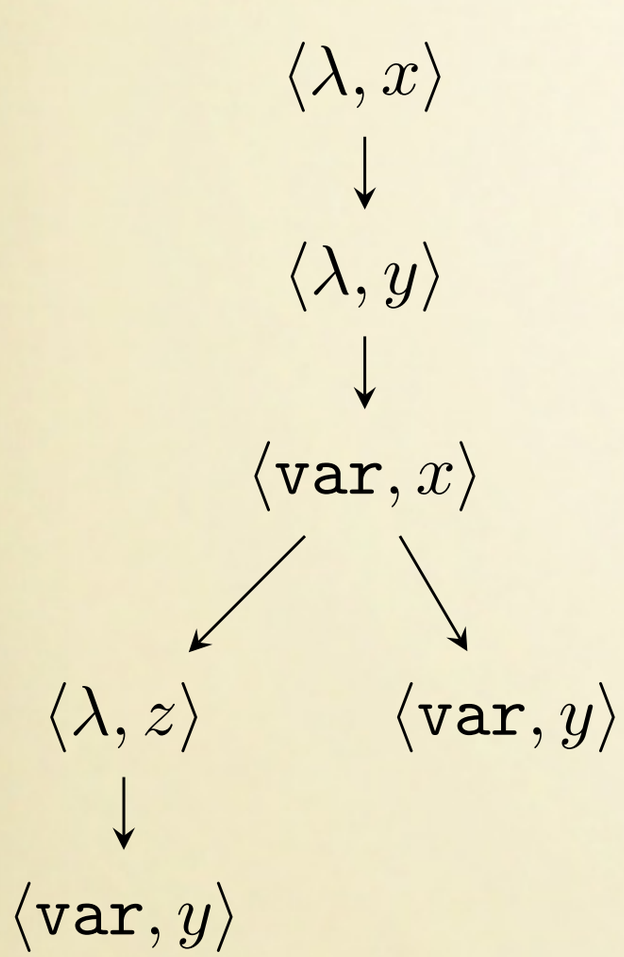
$$(q_1, [\{x\}_9, \{y\}_4, \{z\}_0]), (q_2, [\{x\}_9, \{y\}_4, \emptyset_0])$$

$$\mathtt{var}, q_2, \mathtt{r}_4 \rightsquigarrow \varnothing$$

$$(q_2, [\{x\}_9, \{y\}_4, \emptyset_0])$$

Inhabitation



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$(q_5, [\{x\}_9, \{y\}_4, \emptyset_0])$

$\lambda, q_6, \varnothing \rightsquigarrow q_1, \mathtt{r}_0$

$(q_6, [\{x\}_9, \{y\}_4, \emptyset_0]), (q_2, [\{x\}_9, \{y\}_4, \emptyset_0])$

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ε

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Proposition Given $\alpha \in \mathcal{T}$ and a normal term M,
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Theorem $\mathcal{L}(\mathcal{A}_\alpha)$ is the set of closed normal inhabitants of α.

Principal Inhabitants

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- $M \in \Lambda(\alpha)$ is called a **principal inhabitant** of α if every other type inhabited by M is an instance of α, i.e. α is the most general type that can be assigned to M

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- for principal inhabitants the inhabitation machines are supplied with two additional global registers
- register `id` keeps track of which occurrences of type variables have to be the same
- register `needs` keeps track of which occurrences of compound subtypes have to have the given structure

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Proposition Given type α and a normal term M, then M is a long inhabitant of α if and only if $\mathcal{P}_\alpha$ accepts M with some output $(\texttt{id}', \texttt{needs}')$.

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Definition
$\mathtt{raise}(\alpha, \mathtt{id}, \mathtt{needs})$ denotes the type obtained from α as follows.

- Every occurrence of a subtype β with $\mathtt{n}(\beta) \in \mathtt{needs}$ is substituted by a fresh type variable;
- all occurrences of type variables are given fresh names, in such a way that two occurrences receive the same name if and only if they belong to the same class in $\mathtt{id}$.

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Proposition If $\mathcal{P}_\alpha$ accepts M with output $(\texttt{id}, \texttt{needs})$, then $\texttt{raise}(\alpha, \texttt{id}, \texttt{needs})$ is the principal type of M.

Corollary M is a principal long normal inhabitant of α if and only if $\mathcal{P}_\alpha$ accepts M with some output $(\texttt{id}, \emptyset)$ such that `id` contains one class for each type variable in α, each class containing exactly the identifiers of all occurrences of that type variable.

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Principal Inhabitants

$$((o_0 \to o_1) \to o_2 \to o_3)_9 \to o_4 \to o_5$$

$\lambda xy.x(\lambda z.y)y$ $\quad$ $(\{5 = 3, 1 = 2 = 4\}, \emptyset)$

$$((o \to o') \to o' \to o'') \to o' \to o''$$

$\lambda xy.y$ $\quad$ $(\{5 = 4\}; \{9\})$

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- complexity of the principal inhabitation problem.

Thank You!